**Chapter 2: Probability**

**2.1 Sample Space**

Definition 1 (Sample Space)

The set of all possible outcomes of an experiment, denoted by *S*, is called the **sample space**.

Example 1 (Simply Listing Outcomes of an Experiment)

A die is rolled one time. Find the sample space, *S*.

Example 2 (Using a Tree Diagram to List Outcomes of an Experiment)

A true/false is answered followed by a multiple-choice question that has 5 choices. Find the sample space, *S*.

Example 3 (Using a Table to List Outcomes of an Experiment)

A two dice are rolled. Find the sample space, *S*.

A table with numbers and symbols

Description automatically generated

**2.2 Events**

Definition 2 (Event)

An **event** is a subset of a sample space.

Example 4

A die is rolled one time. Let *A* be the event that the die lands on an even number. Find the event *A.*

Example 5

A true/false question is answered, followed by a multiple-choice question that has 5 choices. Let *B* be the event that a student answers *c* on the multiple-choice question. Find the event *B****.***

Example 6

Two dice are rolled. Let *C* be the event of rolling doubles. Find the event *C*.

**Important Notes Regarding Notation**

* An event having no outcomes, called the empty set, is denoted as
* Events are typically denoted by capital letters at the beginning of the alphabet (A, B, C, etc.)

Definition 3 (Complement)

The **complement** of an event *A* with respect to *S* is the subset of all elements of *S* that are NOT in *A*. We denote the complement of *A* with the symbol .

We sometimes use a **Venn diagram** to show the relationship between different sets.

A blue and green circle with a letter a

Description automatically generated

Example 7

A die is rolled one time. Let *A* be the event that the die lands on an even number. Let *B* be the event that the die lands on a number greater than 2. Find the complement of each event.

Definition 4 (Intersection)

The **intersection** of events *A* and *B* denoted by ­, is the event consisting of all outcomes in both *A* and *B*.

A diagram of a diagram of a different number of circles

Description automatically generated with medium confidence

Definition 5 (Mutually exclusive, or Disjoint)

Events *A* and *B* are said to be **mutually exclusive**, or **disjoint**, if

A green circles with black text

Description automatically generated

Definition 6 (Union)

The **union** of events *A* and *B* denoted , is the event consisting of all outcomes in *A* or in *B****,*** or in both *A* and *B*.

A diagram of two circles

Description automatically generated

Example 8

A die is tossed one time. Consider the following events:

* *A* is the event that an even number occurs on the upface
* *B* is the event that a number greater than 3 occurs on the upface
* *C* is the event that an odd number occurs on the upface

List all the outcomes in the following events:

1. Are *A* and *B* mutually exclusive?
2. Are *A* and *C* mutually exclusive?

**2.3 Counting Sample Points**

Theorem 1 (Multiplication rule)

If an operation can be performed in ways, and if for each of these ways a second operation can be performed in ways, then the two operations can be performed together in ways.

Theorem 2 (Generalized multiplication rule)

This can be extended, so that a sequence of k operations can be performed in ways.

Example 9

How many lunches, consisting of a soup, a sandwich, a dessert, and a drink, are possible if we can select from four soups, three kinds of sandwiches, five desserts, and four drinks?

Example 10

How many even three-digit numbers can be formed from the digits 1, 2, 3, 6, 9 if each digit can be used only once?

**Factorial Notation**

5! = 5(4)(3)(2)(1)

4! = 4(3)(2)(1)

3! = 3(2)(1)

2! = 2(1)

1! = 1

0! = 1

Generally, *n!,* read as “*n factorial*”, is defined as for any non-negative integer *n*.

Definition 7 (Permutation)

**Permutation** is the number of ordered arrangements of *r* objects selected from *n* distinct objects () without replacement. It is given by

The number of permutations of *n* distinct objects is *n!*.

Example 11

List all the permutations of a, b, c.

Example 12

List all permutations of r = 2 objects that can be selected without replacement form the set of n = 4 objects {a, b, c, d}.

Example 13

In one year, three awards (research, teaching, and service) will be given to a class of 25 graduate students in a statistics department. If each student can receive at most one award, how may possible selections are there?

Definition 8 (Circular permutation)

Permutation that occurs by arranging objects in a circle are called **circular permutations.** Two circular permutations are not considered different unless corresponding objects in the two arrangements are preceded or followed by a different object as we proceed in a clockwise direction.

5 people (a, b, c, d, e) are seated around a table. Consider the following two seating arrangements. Are they different?

A diagram of a diagram

Description automatically generated

By considering one person in a fixed position and arranging the others, we have:

Theorem 3

The number of permutations of *n* objects arranged in a circle is (n – 1)!.

Definition 9 (Combinations)

A **combination** is the number of unordered arrangements of *r* objects selected from *n* distinct objects () without replacement. It is given by

Example 14

List all possible combinations of two objects that can be selected from the set {a, b, c, d}.

Example 15

A production lot contains 20 items, of which five are defective. How many ways can:

1. Two defectives be selected out of the 5 defectives in the population?
2. Three good items be selected out of the 15 good items in the population?
3. Five items be selected out of any of the 20 population items?

Example 16

A committee, consisting of two men and one woman, is to be selected from a population of four men and three women. How many such committees can be formed?

Theorem 4 (Permutations for alike objects)

The number of permutations of *n* objects of which are alike, another are alike, …, and a group of are alike is:

Where .

Example 17

Find the number of permutations of aabbbc.

Example 18

A coin is tossed five times. Let *A* be the event that exactly two heads occur in the five tosses. How many outcomes are in event *A*?

Example 19

A sample of ten individuals is selected at random from a population of potential blood donors. Consider the event that the sample contains exactly two individuals with Type A blood, three with Type B, and 5 with type I blood. How many outcomes are contained in this event?